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If the point P be taken outside the circle of the triangle ABC, then $(OL-OP)$ is negative and thus we get the double sign.

$\therefore \triangle'=\frac{1}{2}(R^2 \pm R'^2).\Pi \sin A$. Hence,

$$\frac{\triangle'}{\triangle}=\frac{\pm\frac{1}{2}(R^2-R'^2).\Pi\sin A}{\frac{1}{2}R^2.\Pi\sin 2A}=\pm\frac{(R^2-R'^2}{R^2}\cdot\frac{\Pi\sin A}{\Pi\sin 2A}.$$

Thus the result, as given by the proposer, is not quite right.

He probably misused the well known formula, $\Sigma \sin 2A=4\,\Pi \sin A$.

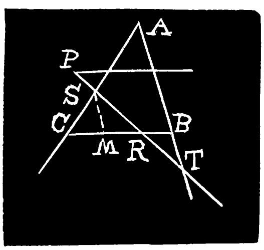
COROLLARY. If the point P lie on the circumscribed circle of triangle ABC, the points X', Y', Z' lie on "Simson's Line" and $\triangle'=0$.

321. Proposed by J. O. MAHONEY, B. E., M. Sc., Central High School, Dallas, Texas.

ABC is an isosceles triangle. Through any point P in its plane draw a line $PSRT$ cutting the sides AC, CB, AB in the points S, R, and T, respectively (R between B and C), so that the segments CS and BT shall be equal.

Solution by C. N. SCHMALL, 89 Columbia Street, New York City.

In the figure, let SM be drawn parallel to AB. Then, if $CS=BT$, $SR=RT$.



Hence to locate the point R we must solve the problem: Given a line drawn through a fixed point, and cutting two fixed intersecting lines, to find the locus of the middle point of the intercepted segment.

Here, we then have $PT-PS=2(PR-PS)$, or $PT+PS=2PR$.

Let (ρ', θ') be the polar co-ordinates of P, and let the equations of AC and AB be

$$a_1x+b_1y+c_1=0\ldots(1),\quad a_2x+b_2y+c_2=0\ldots(2).$$

Transforming to polar co-ordinates these become

$$a_1\rho\cos\theta+b_1\rho\sin\theta+c_1=0\ldots(3),\text{ and } a_2\rho\cos\theta+b_2\rho\sin\theta+c_2=0\ldots(4).$$

From (3), $\rho=-\dfrac{c_1}{a_1\cos\theta+b_1\sin\theta}=PS$; from (4), $\rho=-\dfrac{c_2}{a_2\cos\theta+b_2\sin\theta}=PT$.

Hence the polar equation for the locus of P is, $2\rho=2PR=PT+PS$.

$$\therefore 2\rho=-\frac{c_1}{a_1\cos\theta+b_1\sin\theta}-\frac{c_2}{a_2\cos\theta+b_2\sin\theta}$$

$$-2=\frac{c_1}{a_1\rho\cos\theta+b_1\rho\sin\theta}+\frac{c_2}{a_2\rho\cos\theta+b_2\rho\sin\theta}.$$

Transforming back to rectangular co-ordinates, we have

$$2(a_1x+b_1y)(a_2x+b_2y)+c_1(a_2x+b_2y)+c_2(a_1x+b_1y)=0$$

for the locus of P. This equation represents a hyperbola passing through the vertex A. Hence the intersection of this hyperbola with the base CB will give R, and PR produced will give T.

CALCULUS.

248. Proposed by REV. R. D. CARMICHAEL, Anniston, Ala.

Evaluate $\int_0^{\frac{1}{2}\pi}\sin nx\cot x\,dx$, where n is a positive integer.

II. Solution by FRANCIS RUST, C. E., Pittsburg, Pa.

$$\sin nx=n\cos^{n-1}x\sin x-\binom{n}{3}\cos^{n-3}x\sin^3x+\binom{n}{5}\cos^{n-5}x\sin^5x-\dots$$

$$\therefore\int_0^{\frac{1}{2}\pi}\sin nx\cot x\,dx=n\int_0^{\frac{1}{2}\pi}\cos^n x\,dx-\binom{n}{3}\int_0^{\frac{1}{2}\pi}\cos^{n-2}x\sin^2x\,dx$$

$$+\binom{n}{5}\int_0^{\frac{1}{2}\pi}\cos^{n-4}x\sin^4x\,dx\dots\pm\binom{n}{2r+1}\int_0^{\frac{1}{2}\pi}\cos^{n-2r}x\sin^{2r}x\,dx+\dots$$

Transforming $\int_0^{\frac{1}{2}\pi}\sin^p z\cos^q z\,dz$ by the substitution $\sin z=\sqrt{x}$, we have

$$dz=\frac{dx}{2\sqrt{[x(1-x)]}},\text{ and }\int_0^{\frac{1}{2}\pi}\sin^p z\cos^q z\,dz=\tfrac{1}{2}\int_0^1 x^{\frac{1}{2}(p-1)}(1-x)^{\frac{1}{2}(q-1)}dx$$

$$=\tfrac{1}{2}B[\tfrac{1}{2}(p+1),\tfrac{1}{2}(q+1)].$$

$$\therefore\int_0^{\frac{1}{2}\pi}\sin nx\cot x\,dx,\text{ in beta-functions,}=\frac{n}{2}B\left(\frac{n+1}{2},\frac{1}{2}\right)$$

$$-\tfrac{1}{2}\binom{n}{3}B\left(\frac{n-1}{2},\frac{3}{2}\right)+\tfrac{1}{2}\binom{n}{5}B\left(\frac{n-3}{2},\frac{5}{2}\right)-\dots$$

Also solved by C. E. White.

250. Proposed by V. M. SPUNAR, East Pittsburg, Pa.

Differentiate $(\log^n x)$.